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MATHEMATICAL INVESTIGATION

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Number Theory

1. Eight times table

Below is a calculation. Check it to see if it is correct.

$1 \times 8 + 1 = 9$

$12 \times 8 + 2 = 98$

Continue this calculation set by looking at $123 \times 8 + 3$, $1234 \times 8 + 4$, ... etc. Is there a pattern that emerges? How far can you continue before the pattern breaks down? Does the pattern every break down?

2. Divisibility by nine

If you add up the digits in a number, you get another number called the digit sum. If you add the digits of the digit sum, you get another number. If you continue this process again and again until you end up with a single digit, you have found the unit digit sum.

Do this for the following numbers from the nine times table.

a) 603 b) 891 c) 1548 d) 67833 e) 3821715

What do you notice about the unit sums of these numbers?

Use this information to find if these numbers are multiples of nine.

f) 892,842,903,829,184,837,329 g) 47,291,038,402,840,282,103,800,285,938,291

h) 3,829,449,202,950,004,829,480,849,289,893,208,349,294,023

3. Divisibility by three

Determining whether a number has three as a factor can be done in exactly the same way as determining whether nine is a factor of a particular number. However, the results are slightly different. See if you can make a rule up for determining whether a number is a multiple of three by calculating the unit sum of the following numbers that are in the three times table.

a) 99 b) 105 c) 12 d) 2,460 e) 20,706 f) 1080

State the rule.

Use your rule to determine whether or not the following numbers are multiples of three.

g) 3,829,389,038,482,763,829,173 h) 892,147,907,281,928,371,823,719,282,910,392,104

i) 3,829,401,982,301,928,310 j) 382,849,102,930,192,381,092,382,109,283

k) 23,891,080,098,349,382 l) 4,892,039,482,029,348,100,937,437

m) 4,892,039,482,093,482,030,200 n) 489,302,394,820,030,201,938,492

4. Divisibility by eight

Because 1000 is divisible by eight, then any multiple of 1000 is divisible by eight. So for example, 6000 is divisible by 8 as 6000 ÷ 8 = 750.

This means that when we are looking to determine a test for divisibility by eight, we don't need to consider any digits from the thousands column up as these are always divisible by eight as 1000 ÷ 8 = 125.

In order to find out whether a number has eight as a factor, we only need to prove that the number constructed from the HTU columns has eight as a factor. In other words, a number is divisible by 8 if and only if the last three columns are divisible by 8.

eg. 28,392,**174** is not divisible by 8 as 174 ÷ 8 =21¾.

48,921,092,381,029,831,012,**576** is divisible by 8 as 576 ÷ 8 = 72.

Use this method to determine whether or not the following numbers are divisible by 8.

a) 38,429 b) 493,029 c) 381,939,029,104,784

d) 389,234,298,420,124 e) 849,302,934,801,874

5. Divisibility by eleven

Calculating this is a little more complicated that the factors that we have already come across. You need to split the number into odd and even digits. Once you have done this, you find the difference between the unit sum of these numbers. If the difference between the unit sum of these two numbers is in the eleven times table, you have a number that is divisible by eleven.

eg

1**6**9**3**2**9**7**4**2**8**8**4**9**2** --> (1 + 9 + 2 + 7 + 2 + 8 + 9) - (6 + 3 + 9 + 4 + 8 + 4 + 2) = 38 - 36 = 2 so not divisible by 11.

1**8**5**9**2**7**9**8**2**1**9**1**7**5** --> (1 + 5 + 2 + 9 + 6 + 9 + 7) - (8 + 9 + 7 + 8 + 1 + 1 + 5) = 39 - 39 = 0 so divisible by 11.

6**1**7**3**8**2**9**1**9**2**8**1**9**2** --> (6 + 7+ 8 + 9 + 9 + 8 + 9) - (1 + 3 + 2 + 1 + 2 + 1 + 2) = 56 - 12 = 44 which is divisible by 11.

Which of the following numbers are divisible by eleven?

a) 45,839,203,482,304,832,908,402,937,509,823

b) 4,489,892,019,028,190,002,940,928,111,918,399

c) 89,208,410,102,881,900,291,703,860,574,073,012,054,532,143,048,486,054,100

Sequences

6. Linear Arithmetic Sequences

A sequence of numbers can be represented in the following manner:

n	1	2	3	4	5	6	7	8	9
a_n	12	15	18	21	24	27	30	33	36

The sequence above goes up in threes. I know this because $a_5 - a_4 = 24 - 21 = 3$.

This tells me that $a_n = 3n +$ *something*. 3n means 3 times n.

To work out the *something*, I have to take any of the numbers, eg a_3 which is 18, and work out how much more or less than 3 times a_3 is more than 3 times 3.

So in our a_3 example, a_3 is 18 where 3n or 3×3 =9. 18 - 9 = 9 so the *something* that we are trying to calculate is 9.

So the equation for the sequence above is:

$$a_n = 3n + 9$$

Once I have calculated this equation, I can work out all other members of this sequence by substituting the numbers into the sequence.

So to work out the 250th number in the sequence, we have:

$$a_{250} = 3(250) + 9$$

$$= 750 + 9$$

$$= 759$$

Calculate the following numbers in the sequences.

a) 827th number in the sequence.

b) 82917 number in the sequence.

c) 5391st number in the sequence.

d) 184729 number in the sequence.

Rearrange the equation you have been given so that you can work out what number in the sequence a particular item would fall, for instance, 39 falls tenth in this sequence.

Where would the following numbers fall?

e) 37128372

f) 2819374827

g) 4918392874389921

n	1	2	3	4	5	6	7	8	9
a_n	17	28	39	50	61	72	83	94	105

h) Calculate a formula to give the value of a_n for any value of n.

i) Rearrange the formula to give a value of n for any value of a_n.

Use the formulae to calculate the answers to the following questions.

Give the value of a_n for the following values of n:

j) 482 k) 3829 l) 9382 m) 489320 n) 48923029

Give the value of n for the following values of a_n:

o) 83 p) 1073 q) 26,087 r) 140,707 s) 322,438

Work out the 120th term and the 315th term of the following sequences.

t)

n	1	2	3	4	5	6	7
a_n	14	10	6	2	-2	-6	-10

u)

n	1	2	3	4	5	6	7
a_n	4.5	6	7.5	9	10.5	12	13.5

v)

n	1	2	3	4	5	6	7
a_n	-18	-13	-8	-3	2	7	12

w)

n	1	2	3	4	5	6	7
a_n	-18	-33	-48	-63	-78	-93	-108

7. Special Sequences

There are lots of special sequences which are not linear. That means that they do not go up by the same amount each time.

Leonardo Fibonacci came up with a special sequence which he derived by adding the previous two numbers in the sequence.

$$F_n = F_{n-1} + F_{n-2}$$

The sequence starts like this:

n	1	2	3	4	5	6	7	8	9
F_n	1	1	2	3	5	8	13	21	34

So $F_3 = F_{3-1} + F_{3-2} = F_2 + F_1 = 1 + 1 = 2$,

$F_6 = F_{6-1} + F_{6-2} = F_5 + F_4 = 5 + 3 = 8$ etc.

a) Calculate the first 100 Fibonacci numbers.

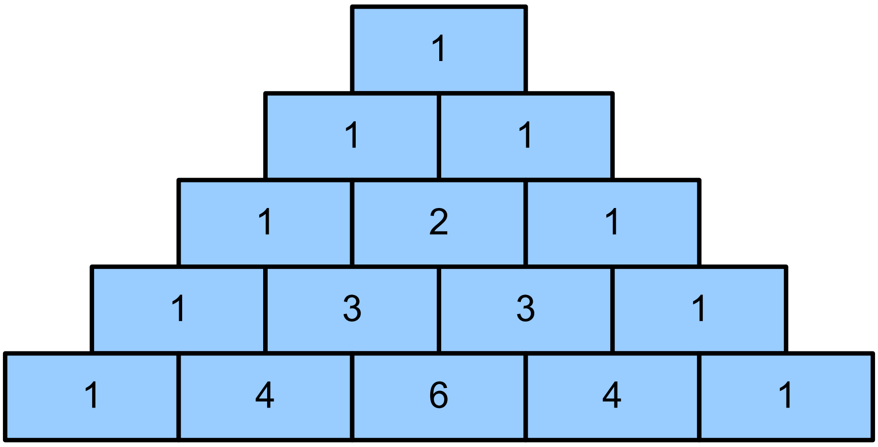
Zeckendorf Theorum states that any whole number can be represented by the addition of any non-consecutive Fibonacci numbers. eg. 100 = 89 + 8 + 3.

b) For each number between 50 and 100, make a sum to demonstrate the Zeckendorf Theorum.

c) Investigate the Fibonacci Sequence by looking at the differences between their squares and the differences between the differences. Is there a pattern of any description? You can use Excel to work these out if you wish.

8. Pascal's Triangle

Look at the figure below. It shows a triangle. Each of the numbers has been calculated by adding together the two numbers above it. At the edge of the triangle, one of the numbers is zero hence the repeated ones.



If we said that $P_1 = \sum \mathit{Numbers\ in\ boxes\ on\ row}\ 1$, then $P_1 = 1$, $P_2 = 2$, $P_3 = 4$, $P_4 = 8$ etc. Can you work out what the following values of P would be equivalent to by adding the numbers on that particular row?

Can you figure out a pattern? Does the pattern hold for all values?

Can you find any other sequences within Pascal's Triangle?

a) What is the value of P_{12} ?

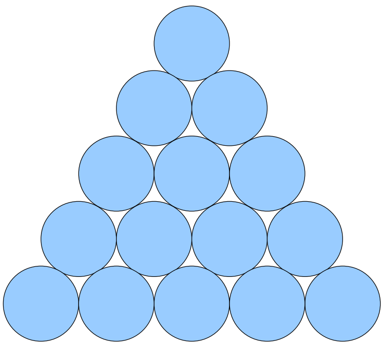
b) What is the value of P_{20}?

c) What is the value of P_{45}?

d) What is the value of P_{100}?

9. Polygonal Numbers

a. Triangular Numbers

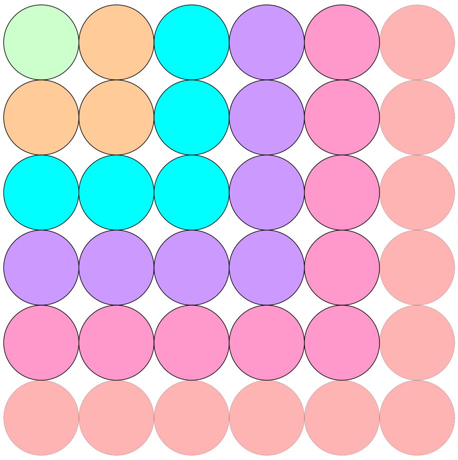


Triangular numbers are formed by adding another row onto the triangle. The difference between the existing triangular number and the next one is the number of circles in the next row.

The sequence begins: 1, 3, 6, 10, 15, 21, 28,...

i) Can you work out a formula to calculate any number within the triangular sequence?

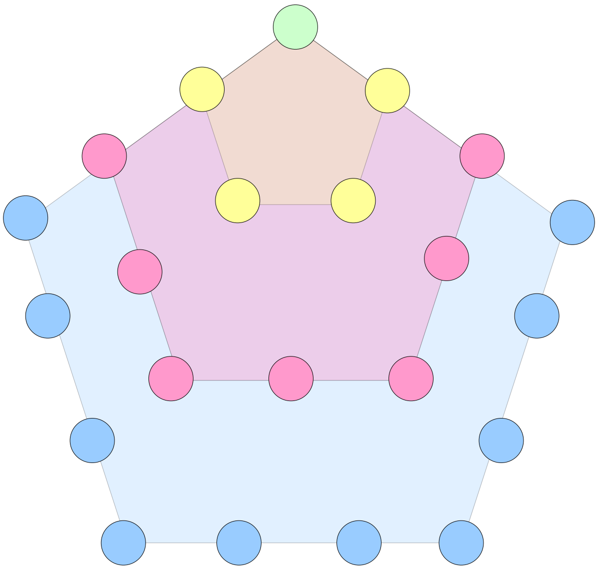
b. Square Numbers



Square numbers are formed by squaring the numbers in a sequence. $1^2, 2^2, 3^2, ..., n^2$.

If you look at the number added each time to the sequence, you might be able to work out a pattern.

c. Pentagonal Numbers



Pentagonal numbers are based around the shape of a pentagon as you can see. The sequence goes like this:

1, 1+4, 1+4+7, 1+4+7+10, 1+4+7+10+13,...

which give:

1,5, 12, 22, 35, ...

Note that each edge of the pentagon has an increasing number of dots upon it.

i) Describe the increase each time in words.
ii) See if you can predict what the 6th, 7th,8th, 9th and 10th terms would be in the sequence.
iii) Can you derive a formula to describe the pentagonal sequence of numbers.

d. Other Polygonal Numbers

There are polygonal numbers associated with every shape. The sequences grow in a similar way to the pentagonal sequence. See if you can work out how octagonal numbers work. The sequence begins in the following way: 1, 8, 21, 40, 65, 96, 133, 176,...

Examine the differences between each number and see if you can work out a pattern.

Mathematical Modelling

If you superimpose a set of squares onto a number grid, can you investigate what happens to the total of the underlying grid?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The above grid demonstrates this using first a 1^2 (in green), then 2^2 (in green and yellow), and finally 3^2 (in green, yellow and blue).

The mathematical modelling process involves you thinking about what could alter the totals. Some of these things are fixed and some are variable. For instance, the grid may be fixed whereas the position of the square upon the grid could alter.

The above example is shown on a grid of size 10 by 10. This is not the only grid size you can use for this model. You might have a grid size of 20 by 20. How will this affect the result you get?

Again, the example above shows a 3 by 3 square. This could be a 4 by 4 square or a 12 by 12 square (on a larger grid). How will this affect the resulting total?

The position on the above grid is such that square is placed in the top left hand corner of the grid. What would happen if instead of using the 1 grid square as a basis, you used the 64th grid square as the basis for your position?

Can you come up with a model that will predict the total of the numbers under the square, no matter what size the square or what size the grid?